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ON TRANSPARENT BOUNDARY CONDITIONS FOR THE HIGH-ORDER HEAT EQUATION

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Abstract. When computing the solution of a partial differential equation in an unbounded domain, one often introduces artificial boundaries. In order to limit the computational cost, these boundaries must be chosen not too far from the domain of interest. Therefore, the boundary conditions must be good approximations to the so-called transparent boundary condition (i.e., such that the solution of the problem in the bounded domain is equal to the solution in the original domain). This question is of crucial interest in such different areas as geophysics, plasma physics, fluid dynamics [2,3,5]. In view of the mentioned shortcomings of the methods described, the need for practical transparent (artificial) boundary conditions combining efficiency and simplicity is evident. Such conditions must satisfy several criteria: (i) The resulting initial boundary value problem should be unique and stable; (ii) the solution to the initial boundary value problem should coincide or closely approximate the solution of the infinite problem on; and (iii) the conditions must allow for an analytical solution or an efficient numerical implementation. In this work we consider artificial boundary conditions for the high-order Cauchy problem for the heat equation. The conditions satisfy the above-mentioned criteria (i), (ii) and (iii). Similar results were taken for the Laplace equation in [4]. The transparent boundary condition is usually an integral relation in time and space between u and its normal derivative on the boundary, which makes it impractical from a numerical point of view. Alternatively, the requirement for boundary conditions can be avoided when the solution of a partial differential equation is approximated in the form of convolution in space and time with the fundamental solution. An efficient approximation of this type for the heat equation is proposed in [1].

Литература

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