Note on the CLT for positively associated random fields

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There are a number of stochastic models involving families of associated random variables or related systems and in the framework of such models the principle limit theorems of Probability theory were established. In this regard we refer, e.g., to [1]. Recall that C.M.Newman [2] proved the classical CLT for associated random fields and formulated the conjecture concerning necessary and sufficient conditions for such a result when the finite susceptibility condition does not hold. N.Herrndorf has shown that this conjecture fails even for associated stochastic processes. We mention also the Vronski [3] theorem containing sufficient conditions for asymptotic normality of a positively associated stationary stochastic process with the finite absolute moment of order $2 + \delta$ ($\delta > 0$). A. Bulinski [4] has demonstrated how one can modify the Newman conjecture to get necessary and sufficient conditions of the CLT validity for positively associated stationary random fields. Our result extends the Vronski theorem [3] to random fields. Namely, we consider a positively associated stationary random field $X = \{X_n, n \in \mathbb{Z}^d\}$ such that, for some $\delta \in (0, 1]$, one has $E|X_0|^{2+\delta} < \infty$. Moreover, we assume that $K_X(n) := \sum\{j: \|j\| \leq n\} \text{Cov}(X_0, X_j), n \in \mathbb{Z}^d_+$, is a very slowly varying function ($\|\cdot\|$ is the sup-norm in $\mathbb{R}^d$). We employ the approach of [4] to prove CLT for that field $X$.

Источники и литература