Секция «Математическая логика, алгебра и теория чисел»

on subword complexity of one sequence

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In this paper, we investigate unipotent dymamics on a torus and apply it to the following problem. For an integer k, consider the sequence of digits $(a_n)_{n>0}$, where an is the first digit in the decimal representation of 2 to the power n^k. For $k = 1$, we study the sequence 1248136125.....

For $k = 2$, we get 12156365121.... and so on. In particular, we are interested in the number of factors of length n that may occur in such a sequence (i.e., the subsequences made of n consecutive digits).

The sequence made by powers of 2 one the circle with irrational angle is dense. It was proved in that case that $p_w(n) = 4n + 5$.

Finally, the last part is dedicated to the very interesting relation existing between k-dimensional torus with the sequence of left-most digit occurring in the decimal representation of 2 to the power n^k, where n, k are positive integers.

Digital problems of this type in Number theory are well-known to be difficult, e.g., in the literature, least non-zero digit of $n!$ in base 12 (Deshouillers et al.) or digits of n^n have been investigated. In particular, this permitted me to be familiarized with notions coming from symbolic dynamics.

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