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Using statistical geometry for partition of hyper planes

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An important class of problems in the theory of geometric transformations is the problem of studying random partitions of space. In the plane case, it is possible to express the distribution of a number of quantities associated with a random partition through an equation in partial derivatives, which in some cases can be reduced to the Riccati equation. The reason for this is that the difficulty associated with the fact that the parts of the partition have and are described by the same unlimited number of parameters, in the planar case, is overcome by considering the secant line. The idea is that a random polygon is interpreted as the evolution of a segment on a moving line. In this case, the work is performed with a limited number of parameters: the area traversed, the meter, etc., the speed of the ends (i.e., the angles of the lines passing through the ends of the section) and the height traversed(the last parameter is necessary for recalculation from the distribution along the second line to the distribution on the plane). When the secant line is shifted, the distribution law does not change, which makes it possible to write a kinetic equation. We will consider the following tasks:

1) A random set of lines is given on the plane, all shifts are equally probable, and the distribution law has the form $F(\varphi)$. What is the distribution of parts of the partition by area? Joint distributions are of interest. The case of the Poisson field of straight lines will be considered in particular.
2) A random set of points is marked on the plane. Each point A is associated with its area of attraction, i.e. the set of points in the plane for which point A is the nearest of the marked ones. The regions of attraction are convex polygons, and the same questions arise for them. In the first problem, denote by $N(S, l, \alpha_1, \alpha_2, t)$ the density of the number of traversed parts adjacent to the line L, with the traversed area S, the length of the trace l, and the angles α_1, α_2 at the ends of the trace, and the lowest vertex of which is at a distance t from L.

$$l\frac{\partial N}{\partial S} + (\operatorname{ctg}\alpha_1 + \operatorname{ctg}\alpha_2)\frac{\partial N}{\partial l} + \frac{\partial N}{\partial t} + \lambda N(\operatorname{tg}\alpha_1/2 + \operatorname{tg}\alpha_2/2) - \lambda\int_0^{\alpha_1} N sin(\alpha_1 - \varphi)/sin(\varphi)d\varphi - \lambda\int_0^{\alpha_2} N sin(\alpha_2 - \varphi)/sin(\varphi)d\varphi = 0. \quad (1)$$

Similarly, you can write out the kinetic equation of the perimeters. Let p be the passed perimeter:

$$(1/\cos\alpha_1 + 1/\cos\alpha_2)\frac{\partial N}{\partial P} + \frac{\partial N}{\partial t} + \frac{\partial N}{\partial l}(\operatorname{ctg}\alpha_1 + \operatorname{ctg}\alpha_2) + \lambda N(\operatorname{tg}\alpha_1/2 + \operatorname{tg}\alpha_2/2) -$$
$$\lambda \int_0^{\alpha_1} N sin(\alpha_1 - \varphi)/sin(\varphi) d\varphi - \lambda \int_0^{\alpha_2} N sin(\alpha_2 - \varphi)/sin(\varphi) d\varphi = 0. \quad (2)$$

This paper will show how to use the Laplace equation to reduce these equations to the Riccati equation.

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