Секция «Математическая логика, алгебра и теория чисел»

The complexity function of leading digits a natural numbers in arbitrary bases

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Consider the sequence of $\mathbf{w} = \{\omega_k\}_{k\geq 0}$, where ω_k is the first digit in the decimal representation of $\mathbf{p}^k$. For $\mathbf{p} = 2$, we study the sequence

$$1248136125124813612512481 36\cdots .$$

In particular, we are interested in the number of factors of length n that may occur in such a sequence (i.e., the subwords made of n consecutive digits). We discussed about complexity function of leading digits of one-digit primes [1].

A *word* over an alphabet $\mathcal{A}$ is a sequence taking values in the finite set $\mathcal{A}$. A *finite word* of length n is thus a map from $\{1, \cdots, n\}$ to $\mathcal{A}$ and a (one-sided) *infinite word* is just a map from $\mathbb{N}$ to $\mathcal{A}$.

Words have a strong representational power: they can encode elements of an infinite set using finitely many symbols, e.g., the characteristic sequence of a subset of integers or the base-d expansion of an irrational number in $[0, 1]$. They naturally appear in a variety of contexts: computability theory, symbolic dynamics, algebra, number theory and numeration systems or theoretical computer science and text algorithms.

It is natural to associate some measures of complexity with infinite words. The most studied one is the *factor complexity* studied in 1975 by Ehrenfeucht, Lee and Rozenberg. It counts the number $C_{\mathbf{w}}(n)$ of distinct factors of length n (blocks of n consecutive letters) occurring in an infinite word $\mathbf{w}$. For instance, ultimately periodic words are characterized, thanks to the Morse-Hedlund theorem, by the fact that $C_{\mathbf{w}}(n)$ is bounded.

The result we have obtained is as follows:

Proposition. Consider the sequence $\{t_n\}_{n\geq 0} = \{a^n\}_{n\geq 0}$. We have the number $a \in (1, +\infty)$ in base $b \in \mathbb{Z}_{\geq 1}$. Then,

1) if $\log_b(a) \in \mathbb{Q}$, then the complexity function is a constant number;
2) if $\log_b(a) \in \mathbb{R} - \mathbb{Q}$, then $C_{a,b}(k) = (b-1)k - m_{a,b}(k-1)$, where $m_{a,b}$ is the number of "collisions".

Источники и литература

1) M. Golafshan, A. Kanel-Belov, A. Heinis and G. Potapov, "On the complexity function of leading digits of powers of one-digit primes,"2021 52nd Annual Iranian Mathematics Conference (AIMC), 2021, pp. 13-15, doi: 10.1109/AIMC54250.2021.9656982.