Почти-вложения графа K_5.

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Denote $S^2 := \{(x, y, z) : \max\{|x|, |y|, |z|\} = 1\}$.

A continuous map $g : K \to S^2$ of a graph K is called an **almost embedding** if $g(\alpha)\cap g(\beta) = \varnothing$ for any two disjoint edges $\alpha, \beta \subseteq K$.

Denote $[n] := \{1, 2, \ldots, n\}$.

Denote by K_5 the complete graph with the vertices set [5]. Denote by (l, k) the edge between vertices l and k in a graph. Denote by the graph obtained from the graph K_5 by deleting the edge $(4, 5)$.

Denote by the complete graph K_3 with the vertices set [3] ordered by cyclic permutation (1 2 3).

Let us define the **linking number** $(\gamma^1, \gamma^0) \in \mathbb{Z}$ of disjoint oriented closed polygonal line γ^1 on S^2 and ordered pair γ^0 of points in $S^2\setminus\gamma^1$. Take an oriented polygonal line L in general position with γ^1 such that ∂L with order inherited from L coincides with γ^0. By (γ^1, γ^0) denote the sum of the signs of the intersection points of L and γ^1. It is well known that the linking number is well defined.

Теорема 1. *For any almost embedding* $g :\to S^2$ *the absolute value of* $(g|, (g(4), g(5)))$ *does not exceed* 3.

It is well-known that for any almost embedding $g :\to S^2$ the absolute value of linking coefficient $(g|, (g(4), g(5)))$ is odd. There are similar results for embeddings of graphs into $\mathbb{R}^3$, see survey [1]. Theorem 1 gives a constraint on the absolute value of the linking coefficient. Theorem 1 disproves conjecture 1.6(a) in the first arXiv version of [2]. We conjecture that for any almost embedding $g :\to S^2$ the absolute value does not exceed 1.

Some theorems in topology of the plane have technical proofs, e.g. Jordan curve theorem and completeness of the van Kampen planarity obstruction in [3]. This work is no exception.

Список литературы

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[3] *Michael J. Pelsmajer, Marcus Schaefer, and Daniel ˇStefankoviˇc.* Removing even crossings. J. Combin. Theory Ser. B, 97(4):489–500, 2007.