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On the chromatic number of slices without monochromatic unit arithmetic progression

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Given a normed space $\mathbb{R}^n_N$ and a subset $\mathcal{M} \subset \mathbb{R}^n$, the *chromatic number* $\chi(\mathbb{R}^n_N, \mathcal{M})$ is the smallest r such that there exists an r-coloring of $\mathbb{R}^n$ with no monochromatic N*-isometric copy* of $\mathcal{M}$. In these terms, $\chi(\mathbb{R}^n_N) = \chi(\mathbb{R}^n_N, I)$, where I is a two-point set.

This notion was most extensively studied especially for the Euclidean spaces $\mathbb{R}^n_2$. The best asymptotic lower and upper bounds for the growing dimension case belong to Raigorodskii [7] and Larman and Rogers [5, 6] : $(1.239 + o(1))^n \le \chi(\mathbb{R}^n_2) \le (3 + o(1))^n$ as $n \to \infty$. The general result for spaces with arbitrary norm is upper bound on this quantity depending only on the dimension: $\chi(\mathbb{R}^n_N) \le (4 + o(1))^n$ as $n \to \infty$. This result was established by Kupavskii[3].

Consider a sequence of positive reals $\lambda_1, \dots, \lambda_k$, given in [4]. We call a set $\{0, \lambda_1, \lambda_1 + \lambda_2, \dots, \sum_{t=1}^{k} \lambda_t\} \subset \mathbb{R}$ a *baton* and denote it by $\mathcal{B}(\lambda_1, \dots, \lambda_k)$. In case $\lambda_1 = \dots = \lambda_k = 1$, i.e., if the set is just a unit arithmetic progression, we simply denote it by $\mathcal{B}_k$ for a shorthand. We consider only *collinear* N-isometric copies of $\mathcal{B}$. When $\mathcal{B} = \mathcal{B}_k$, we call its collinear N-isometric copies *unit arithmetic progressions in* $\mathbb{R}^n_N$ *of length* $k+1$. In [2] was proved that for any normed space $\mathbb{R}^n_N$, there is $k = k(\mathbb{R}^n_N)$ such that $\chi(\mathbb{R}^n_N, \mathcal{B}_k) = 2$.

For positive numbers $h, n \ge 1$ and $e > 0$, we call a set $\mathbb{R}^n_N \times [0, e]^h$ a *slice* and denote it by *Slice(n,k,e)* [1]. Chromatic number of $Slice(n, k, e)$ is finite. Obviously, for any positive e, the inequalities are satisfied

$$\chi(\mathbb{R}^n_N) \le \chi(Slice(n, h, e)) \le \chi(\mathbb{R}^{n+h}_N).$$

Proposition. For any $h, n \ge 1$ and $e > 0$, there is $k = k(Slice(n, h, e))$ such that

$$\chi(Slice(n, h, e), \mathcal{B}_k) = 2.$$

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