Секция «Математическая логика, алгебра и теория чисел»

The arithmetic and complexity function of the most significant digits of α^n

Научный руководитель – Канель-Белов Алексей Яковлевич

Golafshan Mohammadmehdi
Postgraduate
Московский физико-технический институт, Москва, Россия
E-mail: m.golafshan@phystech.edu

1. Introduction

The arithmetic complexity of an infinite word is the function that counts the number of words of a specific length composed of letters in arithmetic progression (and not only consecutive)

In fact, it's a generalization of the complexity function.

2. Prerequisites

Let $\mathcal{A}$ be a nonempty finite set of symbols, which we call an *alphabet*. An element $a \in \mathcal{A}$ is called a *letter*. A *word* over the alphabet $\mathcal{A}$ is a finite sequence of elements of $\mathcal{A}$. In particular, we let ϵ denote the *empty word*. We use the notation $\mathcal{A}^*$ for the set of all finite words, and $\mathcal{A}^+ = A^* \setminus \{\epsilon\}$, and $\mathcal{A}^{\mathbb{N}}$ set of all infinite words.

A word $\mathbf{u}$ is a *factor* (or *subword*) of a word $\mathbf{w}$ if there exist words $\mathbf{p}, \mathbf{q} \in \mathcal{A}^+$ such that $\mathbf{w} = \mathbf{puq}$. In addition, $\mathbf{p}$ is a *prefix* and $\mathbf{q}$ is a *suffix* of $\mathbf{u}$.

Definition 1. The *factor complexity* or *complexity function* of a finite or infinite word $\mathbf{w}$ is the function $n \mapsto \mathrm{P}_{\mathbf{w}}(n)$, which, for each integer n, give the number $\mathrm{P}_{\mathbf{w}}(n)$ of distinct factors of length n in that word.

Definition 2. Let $\mathbf{w} \in \mathcal{A}^{\mathbb{N}}$ such that $\mathbf{w} = a_0 a_1 \cdots a_n \cdots$, where $a_i \in \mathcal{A}$. We call arithmetic closure of W all

$$A(\mathbf{w}) = \{a_i a_{i+d} a_{i+2d} \cdots a_{i+kd} \mid d \geq 1, k \geq 0\}.$$

The arithmetic complexity of $\mathbf{w}$ is the function $a_{\mathbf{w}}$ who has n notmatch the number $a_{\mathbf{w}}(n)$of words in length n in $A(\mathbf{w})$.

According to this definition, in $A(\mathbf{w})$, the parameter k denotes the length of subwords and d is the distance between the letters in the word. For instance, whenever $d = 1$, we can say that $a_{\mathbf{w}}(n) = \mathrm{P}_{\mathbf{w}}(n)$, where $\mathrm{P}_{\mathbf{w}}(n)$ is the complexity function of $\mathbf{w}$.

3. Main parts

Let $\alpha \in \mathbb{R}_{>0} \setminus \{10^x : x \in \mathbb{Q}\} := \mathbb{R}_{\#}$. Then consider $\mathbf{w}_\alpha$ as the word of leading digits of α^n. The main result is as follows:

3.1. Independently

It means arithmetic complexity of leading digits of α is independent from α.

Proposition 1. A_{w_α} is independent from α. Hence, $A_{\mathrm{w}_\alpha} \subset A_{\mathrm{w}_\beta}$ for any $\alpha, \beta \in \mathbb{R}_{\#}$.

3.2. Computing

Proposition 2. $\mathrm{p}_{\mathbf{w}_\alpha} = \theta(n)$ and $a_{\mathbf{w}_\alpha} = \theta(n^3)$.

Acknowledgement

I would like to greatly thank Julien Cassaign, who gave me this interesting research problem and helped me to formulate it when I was a visitor at the CRIM (International Centre Meetings Mathématiques in Marsseile).

References

1) Rigo, M., Formal Languages, Automata and Numeration Systems 1: Introduction to Combinatorics on Words.
2) Golafshan, M., Kanel-Belov, A., Heinis, A. and Potapov, G., 2021, August. On the complexity function of leading digits of powers of one-digit primes. In 2021 52nd Annual Iranian Mathematics Conference (AIMC) (pp. 13-15). IEEE.