Reliability of Correlation Length Measurements

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Characterisation of rough and irregular profiles and surfaces is essential for various branches of science. It has its applications in study of surface growth, crack formation, time series analysis, etc. In most cases, researchers assume that a rough profile or surface is self-affine and might be characterised in terms of fractal geometry.

Scaling behaviour of a fractal curve in two dimensions might be characterised with the interface width ω, lateral correlation length (LCL) ξ, and self-affine exponent. Interface width characterises vertical fluctuations while LCL is a characteristic length scale for them.

There were suggested a variety of LCL definitions. An abscissa of the first near-zero point of AC function, an abscissa of the e^{-1}-point of AC function, etc. These variability of approaches may generate controversial and misleading results. Even more, it was found that the accuracy of statistical measurements significantly depends on profile's size and quality of measurements. Thus, our goal is to estimate the accuracy of methods used to measure LCL on synthetic profiles.

Autocorrelation (AC) function is wedely used to analyse rough profiles; it is given by

$$A(r) = \frac{\langle (h(x_i) - \langle h \rangle)\,(h(x_i + r) - \langle h \rangle) \rangle}{\omega^2}, \tag{1}$$

where $h(x_i)$ is a height of a profile at position x_i, ω is interface width (root-mean-square), $\langle h \rangle$ is the mean height of a profile, and $\langle \dots \rangle$ denotes averaging over the profile of length L.

There are few definitions of LCL for AC function: A1) an abscissa of the e^{-1}-point of AC function: find LCL as $A(\xi) = e^{-1}$; A2) an abscissa of first-near zero point (FNZP) of AC function: find LCL as $A(\xi) = 0.01$; A3) an abscissa of FNZP of the power-law fit; A4) FNZP of the exponential fit to AC function: A5) an abscissa of the e^{-1}-point of the exponential fit to AC function.

Synthetic profiles were generated with the following procedure: a given correlation matrix C is decomposed with the Cholesky decomposition such that $C = SS^T$ where S is a lower triangular matrix, and the upper index T denotes a transpose matrix. Then a set of correlated profiles is generated as $P = SG$ where G is a set of random Gaussian vectors with the zero mean and interface width of one. In this study, we assume a simple exponential decay of correlations with without periodic or deterministic trends. Therefore, the correlation matrix C represents the exponential correlation model, which is defined as $A(r) = exp[-(r/\xi_o)]$ where ξ_o is a characteristic length scale of the exponential decay. Example of a synthetic profile is given on figure 1a.

Иллюстрации

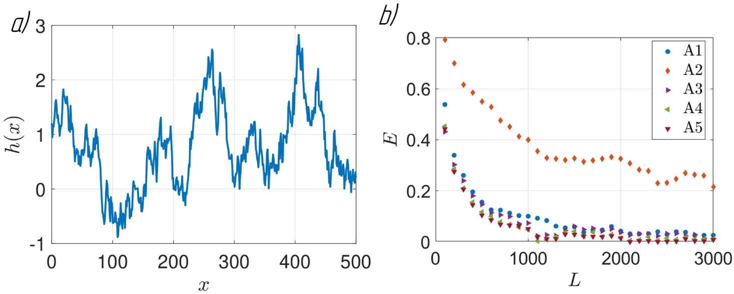

Figure 1: a) An example of a synthetic profile; b) Error estimates

In this study, 100 synthetic profiles with $\xi_o = 10$ were generated and studied. The average LCL, ξ_{med}, was found as median over 100 values; and error of measerements was estimated as a relative error, i.e. $E = |\xi_{med} - \xi_o|/\xi_o$.

Figure 1b quantifies errors of measurements for AC function. It demonstrates that error is decreasing as the profile length is growing for all methods. Also, it shows that method A2 has the lowest convergence rate while the others converge much faster with near the same convergence rate.

Comparison of error curves shows that LCL measurement methods highly depend on length scale of measurements. Generally, error of measurements attenuate as profiles' length is growing.

As the results, we may suggest to use AC function with the definition of LCL as an abscissa of the e^{-1}-point of AC function, method A1. The

AC function is widespread in physics and natively realised in various packages. Moreover, method A1 is simple to code and rapid since no fitting is required; in addition, it has demonstrated high convergence rate.

Литература

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